全固体電池の過渡応答

$$\begin{cases} \dfrac{\partial V_{ae}}{\partial t} = \dfrac{1}{\tau_{ae}} \dfrac{\partial^2 V_{ae}}{\partial x^2}, & \tau_{ae} = (R_{am} + R_{el}) C_{am} \\ & V_{ae} = V_{am} - V_{el} \\ \dfrac{\partial V_{el}}{\partial t} = \dfrac{1}{\tau_{el}} \dfrac{\partial^2 V_{el}}{\partial x^2}, & \tau_{el} = R_{el} C_{am} \end{cases}$$

$$\boxed{\begin{aligned} &\mathcal{L}\left[\frac{\partial V}{\partial t}\right] = s\bar{V} - V(t=0) = \frac{1}{\tau}\frac{d^2\bar{V}}{dx^2} \\ &\therefore \frac{d^2\bar{V}}{dx^2} - s\tau\bar{V} = -\tau V(t=0) \end{aligned}}$$

at $t=0$, $0 \leq x \leq l$.

$$V_{el} = \frac{I^0 R_{el} l}{2}\left(1 - \frac{x^2}{l^2}\right)$$

$$V_{am} = C + \frac{I^0 R_{am} l}{2}\left(1 - \frac{x}{l}\right)^2$$

$$\therefore V_{ae} = C + \frac{I^0 l}{2}\left[(R_{am} + R_{el})\frac{x^2}{l^2} - 2R_{am}\frac{x}{l} + (R_{am} - R_{el})\right]$$

at $t>0$, $x=0$.

$$\left.\frac{\partial V_{ae}}{\partial x}\right|_{x=0} = \left.\frac{\partial V_{el}}{\partial x}\right|_{x=0} = 0,$$

at $t>0$, $x=l$.

$$\left.\frac{\partial V_{ae}}{\partial x}\right|_{x=l} = \left.\frac{\partial V_{el}}{\partial x}\right|_{x=l} = 0, \quad V_{el} = 0.$$

V_{el}について、$I_{el} = ax + C$ とおくと、$x=0$; $I_{el}=0$, $x=l$; $I_{el} = I^0$

$\therefore C=0$, $a = \frac{I^0}{l}$; $I_{el} = \frac{I^0}{l}x$

$\frac{dV_{el}}{dx} = -I_{el}R_{el} = -\frac{I^0}{l}x$; $V_{el} = -\frac{I^0 R_{el} x^2}{2l} + C$.

at $x=l$, $V_{el}=0$,

$\therefore V_{el} = -\frac{I^0 R_{el} l}{2} + C = 0$, $\therefore C = \frac{I^0 R_{el} l}{2}$

$\therefore V_{el} = \frac{I^0 R_{el} l}{2}\left(1 - \frac{x^2}{l^2}\right)$

V_{am}について、$I_{am} = ax + C$とおくと、$x=0$; $I_{am} = I^0$, $x=l$; $I_{am}=0$

$\therefore C = I^0$; $I_{am} = ax + I^0$, $a = -\frac{I^0}{l}$; $I_{am} = I^0\left(1 - \frac{x}{l}\right)$

$\frac{dV_{am}}{dx} = -I_{am}R_{am} = I^0 R_{am}\left(\frac{x}{l} - 1\right)$

$\therefore V_{am} = \frac{I^0 R_{am} l}{2}\left(\frac{x}{l} - 1\right)^2 + C.$

$$\frac{d^2\bar{V}_{e1}}{dx^2} - s\tau_{e1}\bar{V}_{e1} = -\tau_{e1}\frac{I^0 R_{e1} l}{2}\left(1-\frac{x^2}{l^2}\right).$$

$$\frac{d^2\bar{V}_{ae}}{dx^2} - s\tau_{ae}\bar{V}_{ae} = -\tau_{ae}\frac{I^0 l}{2}\left[(R_{am}+R_{e1})\frac{x^2}{l^2} - 2R_{am}\frac{x}{l} + (R_{am}-R_{e1})\right]$$

$(D^2 - s\tau)\bar{V} = 0$ とおくと、 $D = \pm\sqrt{s\tau}$　　∴ $\bar{V}_b = C_1\exp(x\sqrt{s\tau}) + C_2\exp(-x\sqrt{s\tau})$

$\bar{V}_p = ax^2+bx+c$ とおくと．

$$\frac{d^2\bar{V}_p}{dx^2} - s\tau\bar{V}_p = -s\tau a x^2 - s\tau b x + 2a - s\tau c.$$

◎ $\bar{V}_{e1}$ について．

$$-s\tau_{e1}ax^2 - s\tau_{e1}bx + 2a - s\tau_{e1}c = -\tau_{e1}\frac{I^0R_{e1}l}{2} + \tau_{e1}\frac{I^0R_{e1}l}{2}\cdot\frac{x^2}{l^2}$$

∴ $-s\cancel{\tau_{e1}}\cancel{x^2}a = \cancel{\tau_{e1}}\frac{I^0R_{e1}l}{2}\cdot\frac{\cancel{x^2}}{l^2}$, $b=0$, $2a - s\tau_{e1}c = -\tau_{e1}\frac{I^0R_{e1}l}{2} = -\frac{I^0R_{e1}}{ls} - s\tau_{e1}c$

∴ $a = -\frac{I^0R_{e1}}{2ls}$　　　∴ $s\tau_{e1}c = \tau_{e1}\frac{I^0R_{e1}l}{2} - \frac{I^0R_{e1}}{ls}$

∴ $c = \frac{I^0R_{e1}l}{2s} - \frac{I^0R_{e1}}{l\tau_{e1}s^2}$

$$\boxed{∴ \bar{V}_{e1} = C_1\exp(x\sqrt{s\tau_{e1}}) + C_2\exp(-x\sqrt{s\tau_{e1}}) - \frac{I^0R_{e1}}{2ls}x^2 + \frac{I^0R_{e1}l}{2s} - \frac{I^0R_{e1}}{l\tau_{e1}s^2}}$$

◎ $\bar{V}_{ae}$ について．

$\cancel{-}s\cancel{\tau_{ae}}\cancel{x^2}a = \cancel{-\tau_{ae}}\frac{I^0 l}{2}(R_{am}+R_{e1})\frac{\cancel{x^2}}{l^2}$,　$-s\cancel{\tau_{ae}}\cancel{x}b = \cancel{\tau_{ae}}I^0 l R_{am}\frac{\cancel{x}}{l}$,　$2a - s\tau_{ae}c = -\tau_{ae}I^0 l\frac{R_{am}-R_{e1}}{2}$

∴ $a = \frac{I^0(R_{am}+R_{e1})}{2ls}$,　$b = -\frac{I^0R_{am}}{s}$,　$s\tau_{ae}c = \frac{I^0(R_{am}+R_{e1})}{ls} + \tau_{ae}I^0 l\frac{R_{am}-R_{e1}}{2}$

∴ $c = \frac{I^0(R_{am}+R_{e1})}{l\tau_{ae}s^2} + \frac{I^0 l}{2s}(R_{am}-R_{e1})$

$$\boxed{∴ \bar{V}_{ae} = C_3\exp(x\sqrt{s\tau_{ae}}) + C_4\exp(-x\sqrt{s\tau_{ae}}) + \frac{I^0(R_{am}+R_{e1})}{2ls}x^2 - \frac{I^0R_{am}}{s}x + \frac{I^0(R_{am}+R_{e1})}{l\tau_{ae}s^2} + \frac{I^0 l(R_{am}-R_{e1})}{2s}}$$

$$\frac{d\bar{V}_{ae}}{dx} = C_3\sqrt{s\tau_{ae}}\exp(x\sqrt{s\tau_{ae}}) - C_4\sqrt{s\tau_{ae}}\exp(-x\sqrt{s\tau_{ae}}) + \frac{I^0(R_{am}+R_{el})}{\ell s}x - \frac{I^0 R_{am}}{s}$$

$$\therefore \left.\frac{d\bar{V}_{ae}}{dx}\right|_{x=0} = C_3\sqrt{s\tau_{ae}} - C_4\sqrt{s\tau_{ae}} - \frac{I^0 R_{am}}{s} = 0$$

$$\therefore \left.\frac{d\bar{V}_{ae}}{dx}\right|_{x=\ell} = C_3\sqrt{s\tau_{ae}}\exp(\ell\sqrt{s\tau_{ae}}) - C_4\sqrt{s\tau_{ae}}\exp(-\ell\sqrt{s\tau_{ae}}) + \frac{I^0(R_{am}+R_{el})}{s} - \frac{I^0 R_{am}}{s} = 0$$

$$\therefore (C_3 - C_4)\sqrt{s\tau_{ae}} = \frac{I^0 R_{am}}{s}$$

$$\therefore C_3 - C_4 = \frac{I^0 R_{am}}{s\sqrt{s\tau_{ae}}}$$

$$\therefore C_3 = \frac{I^0 R_{am}}{s\sqrt{s\tau_{ae}}} + C_4$$

$$[C_3\exp(\ell\sqrt{s\tau_{ae}}) - C_4\exp(-\ell\sqrt{s\tau_{ae}})]\sqrt{s\tau_{ae}} = -\frac{I^0 R_{el}}{s}$$

$$\therefore C_3\exp(\ell\sqrt{s\tau_{ae}}) - C_4\exp(-\ell\sqrt{s\tau_{ae}}) = -\frac{I^0 R_{el}}{s\sqrt{s\tau_{ae}}}$$

$$\therefore \left(\frac{I^0 R_{am}}{s\sqrt{s\tau_{ae}}} + C_4\right)\exp(\ell\sqrt{s\tau_{ae}}) - C_4\exp(-\ell\sqrt{s\tau_{ae}}) = -\frac{I^0 R_{el}}{s\sqrt{s\tau_{ae}}}$$

$$\therefore C_4[\exp(\ell\sqrt{s\tau_{ae}}) - \exp(-\ell\sqrt{s\tau_{ae}})] = -\frac{I^0 R_{el}}{s\sqrt{s\tau_{ae}}} - \frac{I^0 R_{am}}{s\sqrt{s\tau_{ae}}}\exp(\ell\sqrt{s\tau_{ae}})$$

$$\therefore C_4 = -\frac{I^0}{s\sqrt{s\tau_{ae}}}\,\frac{R_{el} + R_{am}\exp(\ell\sqrt{s\tau_{ae}})}{\exp(\ell\sqrt{s\tau_{ae}}) - \exp(-\ell\sqrt{s\tau_{ae}})}$$

$$\therefore C_3 = \frac{I^0}{s\sqrt{s\tau_{ae}}}\left[R_{am} - \frac{R_{el} + R_{am}\exp(\ell\sqrt{s\tau_{ae}})}{\exp(\ell\sqrt{s\tau_{ae}}) - \exp(-\ell\sqrt{s\tau_{ae}})}\right] = -\frac{I^0}{s\sqrt{s\tau_{ae}}}\,\frac{R_{am}\exp(-\ell\sqrt{s\tau_{ae}}) + R_{el}}{\exp(\ell\sqrt{s\tau_{ae}}) - \exp(-\ell\sqrt{s\tau_{ae}})}$$

$$\therefore \bar{V}_{ae} = -\frac{I^0}{s\sqrt{s\tau_{ae}}}\left\{\frac{[R_{am}\exp(-\ell\sqrt{s\tau_{ae}}) + R_{el}]\exp(x\sqrt{s\tau_{ae}}) + [R_{am}\exp(\ell\sqrt{s\tau_{ae}}) + R_{el}]\exp(-x\sqrt{s\tau_{ae}})}{\exp(\ell\sqrt{s\tau_{ae}}) - \exp(-\ell\sqrt{s\tau_{ae}})}\right\}$$
$$+ \frac{I^0(R_{am}+R_{el})}{2\ell s}x^2 - \frac{I^0 R_{am}}{s}x + \frac{I^0(R_{am}+R_{el})}{\ell\tau_{ae}s^2} + \frac{I^0\ell(R_{am}-R_{el})}{2s}$$

$$\therefore \bar{V}_{ae(x=0)} = -\frac{I^0}{s\sqrt{s\tau_{ae}}}\left\{\frac{R_{am}[\exp(\ell\sqrt{s\tau_{ae}}) + \exp(-\ell\sqrt{s\tau_{ae}})] + 2R_{el}}{\exp(\ell\sqrt{s\tau_{ae}}) - \exp(-\ell\sqrt{s\tau_{ae}})}\right\} + \frac{I^0(R_{am}+R_{el})}{\ell\tau_{ae}s^2} + \frac{I^0\ell(R_{am}-R_{el})}{2s}$$

$$= -\frac{I^0}{s\sqrt{s\tau_{ae}}}\left[R_{am}\coth(\ell\sqrt{s\tau_{ae}}) + R_{el}\,\mathrm{cosech}(\ell\sqrt{s\tau_{ae}})\right] + \frac{I^0(R_{am}+R_{el})}{\ell\tau_{ae}s^2} + \frac{I^0\ell(R_{am}-R_{el})}{2s}$$

$$\coth(z) = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{1}{z^2+(n\pi)^2}, \quad \mathrm{cosech}(z) = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{(-1)^n}{z^2+(n\pi)^2}$$

$$\therefore \coth(\ell\sqrt{s\tau_{ae}}) = \frac{1}{\ell\sqrt{s\tau_{ae}}} + 2\ell\sqrt{s\tau_{ae}}\sum_{n=1}^{\infty}\frac{1}{\ell^2\tau_{ae}s + (n\pi)^2} = \frac{1}{\ell\sqrt{s\tau_{ae}}} + \frac{2}{\ell}\sqrt{\frac{s}{\tau_{ae}}}\sum_{n=1}^{\infty}\frac{1}{s + \frac{\pi^2 n^2}{\ell^2\tau_{ae}}}$$

$$\mathrm{cosech}(\ell\sqrt{s\tau_{ae}}) = \frac{1}{\ell\sqrt{s\tau_{ae}}} + 2\ell\sqrt{s\tau_{ae}}\sum_{n=1}^{\infty}\frac{(-1)^n}{\ell^2\tau_{ae}s + (n\pi)^2} = \frac{1}{\ell\sqrt{s\tau_{ae}}} + \frac{2}{\ell}\sqrt{\frac{s}{\tau_{ae}}}\sum_{n=1}^{\infty}\frac{(-1)^n}{s + \frac{\pi^2 n^2}{\ell^2\tau_{ae}}}$$

$$\therefore \bar{V}_{ae}(x=0) = -\frac{2I^0}{l\tau_{ae}s}\left[R_{am}\sum_{n=1}^{\infty}\frac{1}{s+\frac{\pi^2n^2}{l^2\tau_{ae}}} - R_{el}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{s+\frac{\pi^2n^2}{l^2\tau_{ae}}}\right] + \frac{I^0 l(R_{am}-R_{el})}{2s}$$

$$\frac{A}{s}+\frac{B}{s+\frac{\pi^2n^2}{l^2\tau_{ae}}} = \frac{(A+B)s+A\frac{\pi^2n^2}{l^2\tau_{ae}}}{s\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} = \frac{1}{s\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} \qquad \sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}, \quad \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}=\frac{\pi^2}{12}$$

$$\therefore A=-B, \quad A=\frac{l^2\tau_{ae}}{\pi^2n^2}, \quad B=-\frac{l^2\tau_{ae}}{\pi^2n^2}$$

$$\therefore \bar{V}_{ae}(x=0) = -\frac{2I^0}{l\tau_{ae}}\left\{R_{am}\frac{l^2\tau_{ae}}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{1}{sn^2}-\frac{1}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)}\right] - R_{el}\frac{l^2\tau_{ae}}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n-1}}{sn^2}-\frac{(-1)^{n-1}}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)}\right]\right\} + \frac{I^0 l(R_{am}-R_{el})}{2s}$$

$$= -\frac{2I^0}{\cancel{l\tau_{ae}}}\cdot\frac{l^{\cancel{2}}\cancel{\tau_{ae}}}{\pi^2}\left\{\frac{R_{am}}{s}\sum_{n=1}^{\infty}\frac{1}{n^2} - R_{am}\sum_{n=1}^{\infty}\frac{1}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} - \frac{R_{el}}{s}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2} + R_{el}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)}\right\} + \frac{I^0 l(R_{am}-R_{el})}{2s}$$

$$= -I^0 l\frac{2}{\pi^2}\left\{\frac{R_{am}}{s}\sum_{n=1}^{\infty}\frac{1}{n^2} - \frac{R_{el}}{s}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2} - R_{am}\sum_{n=1}^{\infty}\frac{1}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} + R_{el}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)}\right\} + \frac{I^0 l(R_{am}-R_{el})}{2s}$$

$$= -I^0 l\frac{2}{\pi^2}\left\{\frac{\pi^2}{6}\frac{R_{am}}{s} - \frac{\pi^2}{12}\frac{R_{el}}{s} - R_{am}\sum_{n=1}^{\infty}\frac{1}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} + R_{el}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)}\right\} + \frac{I^0 l(R_{am}-R_{el})}{2s}$$

$$= \frac{I^0 l R_{el}}{6s} - \frac{I^0 R_{am} l}{3s} + I^0 R_{am} l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} - I^0 R_{el} l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} + \frac{I^0 l(R_{am}-R_{el})}{2s}$$

$$= I^0 R_{am} l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} - I^0 R_{el} l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2\left(s+\frac{\pi^2n^2}{l^2\tau_{ae}}\right)} + \frac{I^0 l}{s}\left(\frac{R_{am}-2R_{el}}{6}\right)$$

$$\boxed{\therefore V_{ae}(x=0) = I^0 R_{am} l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-n^2\frac{\pi^2 t}{l^2\tau_{ae}}\right) - I^0 R_{el} l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}\exp\left(-n^2\frac{\pi^2 t}{l^2\tau_{ae}}\right) + I^0 l\left(\frac{R_{am}-2R_{el}}{6}\right)}$$

$$\frac{\tau_{ae}}{\tau_{el}} = \frac{R_{am}}{R_{el}}-1 \qquad \therefore R_{am} = R_{el}\left(\frac{\tau_{ae}}{\tau_{el}}-1\right)$$

$\bar{V}_{el}$ について.

$$\bar{V}_{e1} = C_1 \exp(x\sqrt{s\tau_{e1}}) + C_2 \exp(-x\sqrt{s\tau_{e1}}) - \frac{I^0 R_{e1}}{2ls}x^2 + \frac{I^0 R_{e1} l}{2s} - \frac{I^0 R_{e1}}{ls^2\tau_{e1}}$$

$$\frac{d\bar{V}_{e1}}{dx} = C_1\sqrt{s\tau_{e1}}\exp(x\sqrt{s\tau_{e1}}) - C_2\sqrt{s\tau_{e1}}\exp(-x\sqrt{s\tau_{e1}}) - \frac{I^0 R_{e1}}{ls}x$$

$$\left.\frac{d\bar{V}_{e1}}{dx}\right|_{x=0} = (C_1 - C_2)\sqrt{s\tau_{e1}} = 0. \qquad \therefore C_1 = C_2.$$

$$\therefore \frac{d\bar{V}_{e1}}{dx} = C_1\sqrt{s\tau_{e1}}\left[\exp(x\sqrt{s\tau_{e1}}) - \exp(-x\sqrt{s\tau_{e1}})\right] - \frac{I^0 R_{e1}}{ls}x$$

$$\bar{V}_{el} = C_1\left[\exp(x\sqrt{s\tau_{e1}}) + \exp(-x\sqrt{s\tau_{e1}})\right] - \frac{I^0 R_{e1}}{2ls}x^2 + \frac{I^0 R_{e1} l}{2s} - \frac{I^0 R_{e1}}{ls^2\tau_{e1}}$$

$$\left.\frac{d\bar{V}_{el}}{dx}\right|_{x=l} = C_1\sqrt{s\tau_{e1}}\left[\exp(l\sqrt{s\tau_{e1}}) - \exp(-l\sqrt{s\tau_{e1}})\right] - \frac{I^0 R_{el}}{s} = 0.$$

$$\therefore C_1 = \frac{I^0 R_{e1}}{s\sqrt{s\tau_{e1}}} \cdot \frac{1}{\exp(l\sqrt{s\tau_{e1}}) - \exp(-l\sqrt{s\tau_{e1}})}$$

$$\therefore \bar{V}_{e1} = \frac{I^0 R_{e1}}{s\sqrt{s\tau_{e1}}}\,\frac{\exp(x\sqrt{s\tau_{e1}}) + \exp(-x\sqrt{s\tau_{e1}})}{\exp(l\sqrt{s\tau_{e1}}) - \exp(-l\sqrt{s\tau_{e1}})} - \frac{I^0 R_{e1}}{2ls}x^2 + \frac{I^0 R_{e1} l}{2s} - \frac{I^0 R_{el}}{ls^2\tau_{el}}$$

$$\bar{V}_{e1}(x=0) = \frac{I^0 R_{e1}}{s\sqrt{s\tau_{el}}}\operatorname{cosech}(l\sqrt{s\tau_{e1}}) + \frac{I^0 R_{el} l}{2s} - \frac{I^0 R_{el}}{ls^2} \qquad \boxed{\operatorname{cosech}(z) = \frac{1}{z} + 2z\sum_{n=1}^{\infty}\frac{(-1)^n}{z^2 + (n\pi)^2}}$$

$$= \frac{I^0 R_{el}}{s\sqrt{s\tau_{e1}}}\left(\frac{1}{l\sqrt{s\tau_{e1}}} + 2l\sqrt{s\tau_{e1}}\sum_{n=1}^{\infty}\frac{(-1)^n}{l^2 s\tau_{el} + (n\pi)^2}\right) + \frac{I^0 R_{el} l}{2s} - \frac{I^0 R_{e1}}{ls^2\tau_{e1}}$$

$$= \cancel{\frac{I^0 R_{el}}{ls^2\tau_{e1}}} + \frac{2I^0 R_{el}}{l\tau_{e1}s}\sum_{n=1}^{\infty}\frac{(-1)^n}{s + \frac{\pi^2}{l^2\tau_{e1}}n^2} + \frac{I^0 R_{el} l}{2s} - \cancel{\frac{I^0 R_{el}}{ls^2\tau_{e1}}}$$

$$= \frac{2I^0 R_{e1}}{l\tau_{e1}s}\sum_{n=1}^{\infty}\frac{(-1)^n}{s + \frac{\pi^2}{l^2\tau_{e1}}n^2} + \frac{I^0 R_{e1} l}{2s}$$

$$\frac{A}{s} + \frac{B}{s + \frac{\pi^2}{l^2\tau_{el}}n^2} = \frac{1}{s\left(s + \frac{\pi^2}{l^2\tau_{el}}n^2\right)} = \frac{A\left(s + \frac{\pi^2}{l^2\tau_{el}}n^2\right) + Bs}{s\left(s + \frac{\pi^2}{l^2\tau_{el}}n^2\right)} = \frac{(A+B)s + A\frac{\pi^2}{l^2\tau_{el}}n^2}{s\left(s + \frac{\pi^2}{l^2\tau_{el}}n^2\right)}$$

$$\therefore A = -B, \quad A = \frac{l^2\tau_{el}}{\pi^2 n^2}, \quad B = -\frac{l^2\tau_{el}}{\pi^2 n^2}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}=\frac{\pi^2}{12}$$

$$\therefore \overline{V_{e1}}(x=0)=\frac{2I^0R_{e1}}{\cancel{\ell\tau_{e1}}}\cdot\frac{\ell^2\tau_{e1}}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\left(\frac{1}{S}-\frac{1}{S+\frac{\pi^2}{\ell^2\tau_{e1}}n^2}\right)+\frac{I^0R_{e}\ell}{2S}$$

$$=\frac{-I^0R_{e}\ell}{S}\cdot\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}-I^0R_{e1}\ell\frac{2}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{(-1)^n}{n^2}\cdot\frac{1}{S+\frac{\pi^2}{\ell^2\tau_{e1}}n^2}\right)+\frac{I^0R_{e}\ell}{2S}$$

$$=-\frac{I^0R_{e1}\ell}{S}\frac{\cancel{2}}{\cancel{\pi^2}}\frac{\cancel{\pi^2}}{\cancel{12}6}+I^0R_{e1}\ell\frac{2}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{(-1)^{n-1}}{n^2}\cdot\frac{1}{S+\frac{\pi^2}{\ell^2\tau_{e}}n^2}\right)+\frac{I^0R_{e}\ell}{2S}$$

$$=I^0R_{e}\ell\left\{\frac{2}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n-1}}{n^2}\cdot\frac{1}{S+\frac{\pi^2}{\ell^2\tau_{e}}n^2}\right]+\frac{1}{2S}-\frac{1}{6S}\right\}$$

$$=I^0R_{e}\ell\left\{\frac{2}{\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n-1}}{n^2}\cdot\frac{1}{S+\frac{\pi^2}{\ell^2\tau_{e}}n^2}\right]+\frac{1}{3S}\right\}$$

$$\boxed{\therefore V_{e1}(x=0)=I^0R_{e1}\ell\left\{\frac{1}{3}+\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}\exp\left(-\frac{\pi^2 t}{\ell^2\tau_{e}}n^2\right)\right\}}$$

しかしながら、この $V_{e1}(x=0)$ は、$x=0$ と $x=\ell$ での $\frac{dV_{e1}}{dx}=0$ とした場合の式であり、$V_{e1}(x=\ell)$ を規準とした電位とは限らないので、同条件での $V_{e1}(x=\ell)$ を求め、これを規準としなければならない。

$$\therefore \overline{V_{e1}}(x=\ell)=\frac{I^0R_{e1}}{S\sqrt{S\tau_{e1}}}\frac{\exp(\ell\sqrt{S\tau_{e1}})+\exp(-\ell\sqrt{S\tau_{e1}})}{\exp(\ell\sqrt{S\tau_{e1}})-\exp(-\ell\sqrt{S\tau_{e1}})}-\cancel{\frac{I^0R_{e}\ell^2}{2\ell S}}+\cancel{\frac{I^0R_{e1}\ell}{2S}}-\frac{I^0R_{e1}}{\ell\tau_{e1}S^2}$$

$$=\frac{I^0R_{e1}}{S\sqrt{S\tau_{e1}}}\coth(\ell\sqrt{S\tau_{e1}})-\frac{I^0R_{e1}}{\ell\tau_{e1}S^2}$$

$$\coth(\ell\sqrt{S\tau_{e1}})=\frac{1}{\ell\sqrt{S\tau_{e1}}}+\frac{2}{\ell}\sqrt{\frac{S}{\tau_{e1}}}\sum_{n=1}^{\infty}\frac{1}{S+\frac{\pi^2n^2}{\ell^2\tau_{e1}}}$$

$$=\cancel{\frac{I^0R_{e1}}{\ell\tau_{e1}S^2}}+\frac{2I^0R_{e1}}{\ell\tau_{e1}S}\sum_{n=1}^{\infty}\frac{1}{S+\frac{\pi^2n^2}{\ell^2\tau_{e1}}}-\cancel{\frac{I^0R_{e1}}{\ell\tau_{e1}S^2}}$$

$$=\frac{2I^0R_{e1}}{\ell\tau_{e1}}\sum_{n=1}^{\infty}\frac{\ell^2\tau_{e1}}{\pi^2n^2}\left(\frac{1}{S}-\frac{1}{S+\frac{\pi^2n^2}{\ell^2\tau_{e1}}}\right)=\frac{I^0R_{e}\ell}{S}\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}-I^0R_{e1}\ell\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\frac{1}{S+\frac{\pi^2n^2}{\ell^2\tau_{e1}}}$$

$$=\frac{I^0R_{e1}\ell}{3S}-I^0R_{e1}\ell\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\cdot\frac{1}{S+\frac{\pi^2n^2}{\ell^2\tau_{e1}}}$$

$$\boxed{\therefore V_{e1}(x=\ell)=\frac{I^0R_{e1}\ell}{3}-I^0R_{e1}\ell\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-n^2\frac{\pi^2t}{\ell^2\tau_{e1}}\right)}$$

$$\boxed{\therefore V_{e1}'(x=0)=I^0R_{e1}\ell\frac{2}{\pi^2}\left[\sum_{n=1}^{\infty}\frac{1}{n^2}\exp\left(-n^2\frac{\pi^2t}{\ell^2\tau_{e1}}\right)+\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}\exp\left(-n^2\frac{\pi^2t}{\ell^2\tau_{e1}}\right)\right]}$$

以上より、$V_{e1}(x=l)$を電位規準とした$V_{am}(x=0)$の過渡応答は次式となる。

$$V_{am}=V_{ae}+V_{e1}'$$

$$=I^0R_{am}l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{ae}}\right)}{n^2}+I^0R_{e1}l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{e1}}\right)}{n^2}+I^0R_{e1}l\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}\left[\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{e1}}\right)-\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{ae}}\right)\right]$$

$$+I^0l\frac{R_{am}-2R_{e1}}{6}$$ ← 無視する。

$$\frac{V_{am}}{I^0R_{e1}l}=F_{SETHL}$$

$$=\left(\frac{\tau_{ae}}{\tau_{e1}}-1\right)\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{ae}}\right)}{n^2}+\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{e1}}\right)}{n^2}+\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}\left[\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{e1}}\right)-\exp\left(-n^2\frac{\pi^2t}{l^2\tau_{ae}}\right)\right]$$